TITLE:
 Towards an online matching mechanism for kidney paired donation

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BACKGROUND:
        Kidney paired donation (KPD) matches a recipient and his incompatible donor to another pair with a complementary incompatibility, such that the donor of the first pair gives to the recipient of the second and vice-versa. Recipients and their incompatible donors arrive one at a time seeking paired donation, but in order to use optimized matching, programs must require patients to wait many weeks or months for pairs to accumulate. On the other hand, if all recipients were instead matched immediately as they arrived, then the number of transplants achieved would be reduced by at least 20%, and the expected lifespan of the transplants would also suffer (1).
        We propose a radical alternative to our own optimization model of matching for paired donation. We will consider paired donation as a Markov Decision Process (MDP), a mathematical model for making a sequence of decisions in situations that evolve over time. Markov Decision Processes have been successfully employed, for example, to help liver recipients choose the best timing for their living donor transplantation (2). By calculating the best policy (sequence of decisions) for a kidney paired donation MDP, we can allow some, but not all, feasible paired donation matches to proceed to transplant immediately, while preserving the higher match rates achieved by optimization. Our results would offer clinical guidance, tailored to diverse KPD programs which differ in arrival rates and registry composition, about whether and which arriving incompatible pairs should wait before matching.
        The types of matches which should proceed immediately to transplant are those in which at least one of the pairs is a difficult-to-match pair. In contrast, pairs that are easier to match might be better reserved for a later arrival who will be looking for that particular needle in a haystack. Also, as large numbers of pairs accumulate in the pool, it will become more advantageous to match pairs immediately rather than add them to the pool. A realtime allocation method that does not require uniformly long waiting periods for incompatible pairs makes paired donation less cumbersome and expensive, and improves outcomes.

OBJECTIVE:
        Using an MDP model of paired donation, we aim to resolve the quandary faced by every registry - deciding whether arriving incompatible pairs should be matched immediately or should wait to facilitate a greater number of transplants through optimization. The best MDP policy answers the question specifically for each patient and each program. Our specific objectives were to define clinically which incompatible pairs are easy, moderately easy, or difficult to match, to calculate best policies for Markov Decision Processes representing various program sizes, and to evaluate realtime MDP-based matching against an optimized matching benchmark, on criteria including average waiting times and proportion of incompatible pairs transplanted.

METHOD AND RESULTS:
        We categorize the pairs as easy, moderately easy, or difficult to match, based on the probability that the recipient of a pair will be compatible with the donor of another incompatible pair chosen randomly. The probability of finding a match for easy, moderate, and difficult to match pairs are $p(e)$, $p(m)$, and $p(d)$. The MDP state includes three pieces of information: the set of

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unmatched pairs already accumulated in the registry; whether the most recently arrived pair (the on-deck pair) is e, m, or d; and the most difficult to match pair from the pool that matches the on-deck pair. For instance, if the on-deck pair is an m-pair, and there are 5 e-, 2 m-, and 4 d-pairs waiting, and the on-deck pair finds a match to an e-pair but not to an m- or d-pair, we write the state as s=[5,2,4,m,e]. If there are no compatible pairs at all for the on-deck pair, then the last component will be n. The probability of e-, m-, and d-type arrivals, respectively, is f(e), f(m), and f(d).

Each newly arrived pair may either join the pool of accumulated pairs, or proceed with paired donation, if possible, using the most difficult to match pair that is mutually compatible. Depending on the decision made, the accumulation numbers are updated and then a new pair arrives, as illustrated in the partial state transition diagram below:
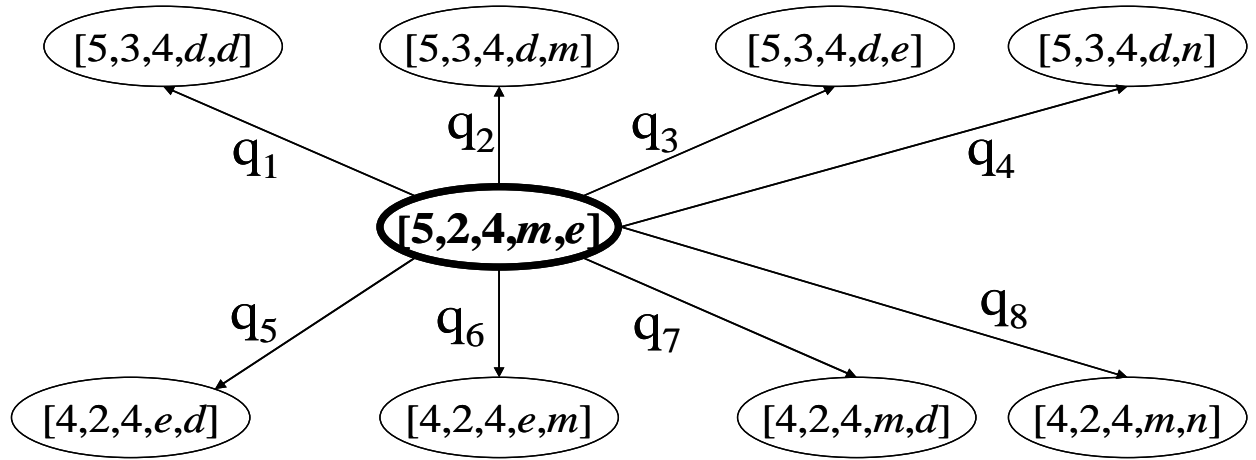
The current state is shown in the bold center oval. The transitions pointing down are transitions in which the on-deck pair is matched to an e-type pair. The transitions pointing up are transitions in which the on-deck pair was added to the pool. Only 8 of 24 transitions are pictured. The number of pairs in the pool will determine the probability of making each of these transitions. For instance, in the diagram above, $q_3 = f(d)[1-p^2(d)]^4[1-p(d)p(m)]^3[1-(1-p(d)p(e))^5]$ if the pair waits, and $q_5 = f(e)[1-(1-p(e)p(d))^4]$ if the pair is transplanted immediately.

For technical reasons, we must discount the objective function by a factor of λ, $0<\lambda<1$. Using λ =1 would mean that the value of receiving a kidney transplant now is exactly the same as the value of receiving a kidney transplant after some waiting period. Since dialysis lowers quality of life and patients incur an incremental risk of dying with delay, must be less than 1. We tune to reflect different incompatible pair arrival rates at different KPD programs. We used MATLAB and the Markov Decision Process Toolbox on our laboratory Linux cluster computer to find the best policies for kidney paired donation MDP models.

The MDP policy accumulated some easy and medium pairs, even when those pairs could have been matched immediately, giving difficult-to-match pairs a greater chance of finding a match. In a detailed clinical simulation, we found that the MDP-derived policy could match up to 94.8% of the maximum number of transplants, while reducing wait time by 58% in some cases. Transplanting immediately, a naive strategy, matched between 86.3-91.1% of the pairs that the optimization method could match, but yielded lower wait times, 69-99.7% lower.

	# Transplanted out of 100	Wait Time Easy Pairs	Wait Time Medium Pairs	Wait Time Difficult Pairs
MDP	39.68 – 41.84	39.97 – 49.96	47.16 – 48.56	27.5 – 29.5
Transplant immediately	37.92 – 40.18	.13 – .95	7.98 – 8.29	17.99 – 18.34
Optimization	41.94 – 44.32	50 – 99	41.83 – 80.57	55.44 – 57.07

The MDP model operates on a necessarily simplified model but we validated this in a clinically detailed simulation against the two alternative methods for matching. By maximizing transplantation of difficult-to-match pairs while minimizing waiting time, this strategy offers a middle-of-the-road option for KPD matching.

PUBLICATIONS:
In preparation.

PRESENTATIONS:

March 2, 2009. Influencing policy with simulation; increasing transplantation with optimization. *Society for Industrial and Applied Mathematicians Conference on Computational Science and Engineering*, Miami FL. Sommer Gentry.

June 1, 2009. The Markov Decision Process can help maximize benefits in paired kidney donation. *American Transplant Congress*, Boston MA. Ann D. Smith, Robert C. Harland, Giuliano Testa, Sommer Gentry.

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